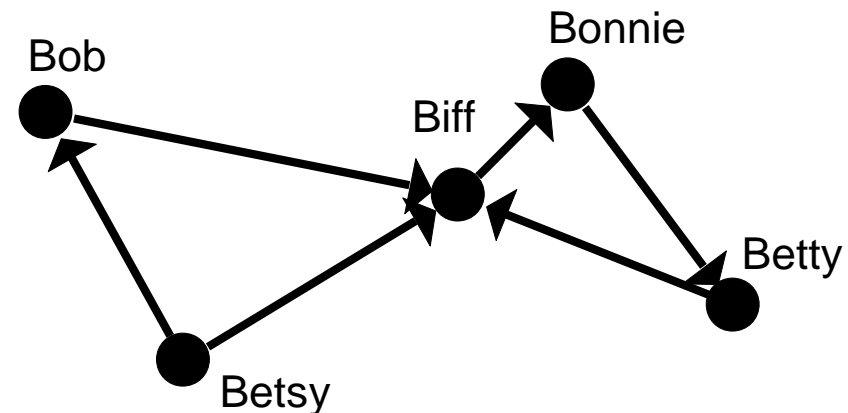
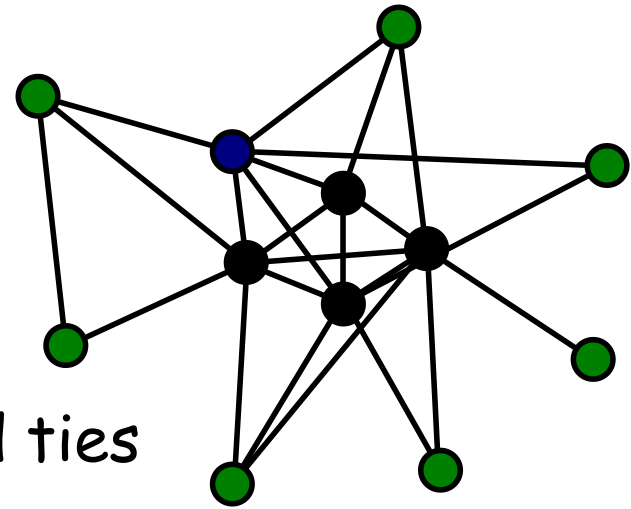


# Graph Theoretic Concepts

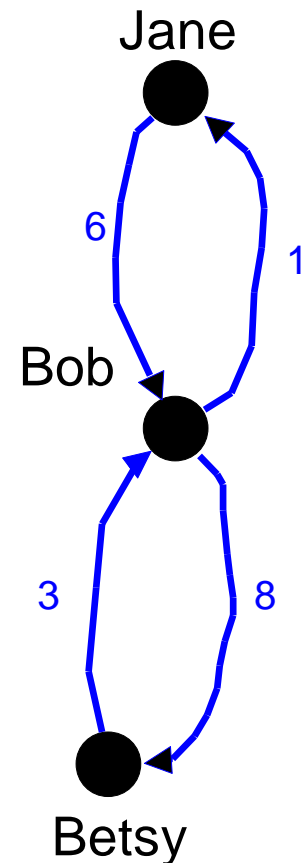
# Directed vs undirected ties

- Undirected relations
  - Attended meeting with
  - Communicates daily with
- Directed relations
  - Lent money to
- Logically vs empirically directed ties
  - Empirically, even undirected relations can be non-symmetric due to measurement error



# Strength of Tie

- We can attach values to ties, representing quantitative attributes
  - Strength of relationship
  - Information capacity of tie
  - Rates of flow or traffic across tie
  - Distances between nodes
  - Probabilities of passing on information
  - Frequency of interaction



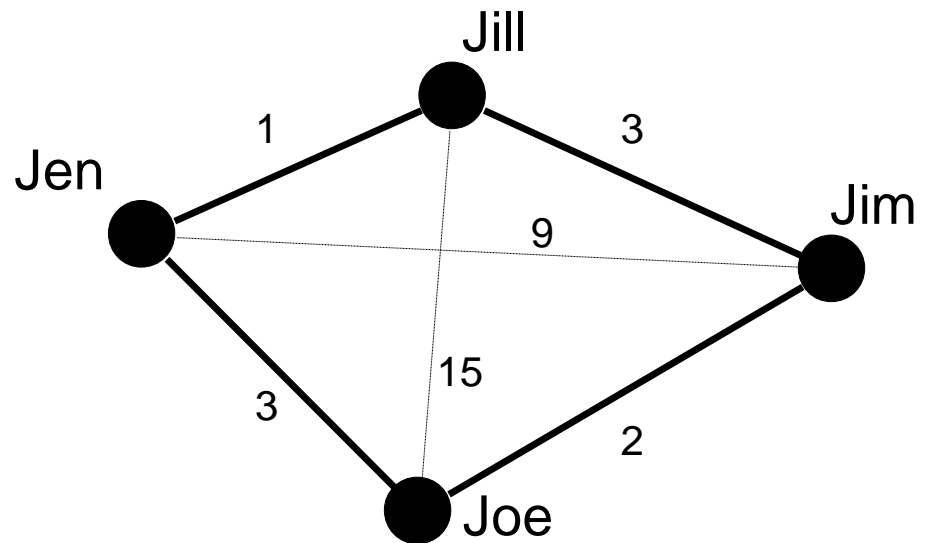
# Adjacency Matrices

Friendship

	Jim	Jill	Jen	Joe
Jim	-	1	0	1
Jill	1	-	1	0
Jen	0	1	-	1
Joe	1	0	1	-

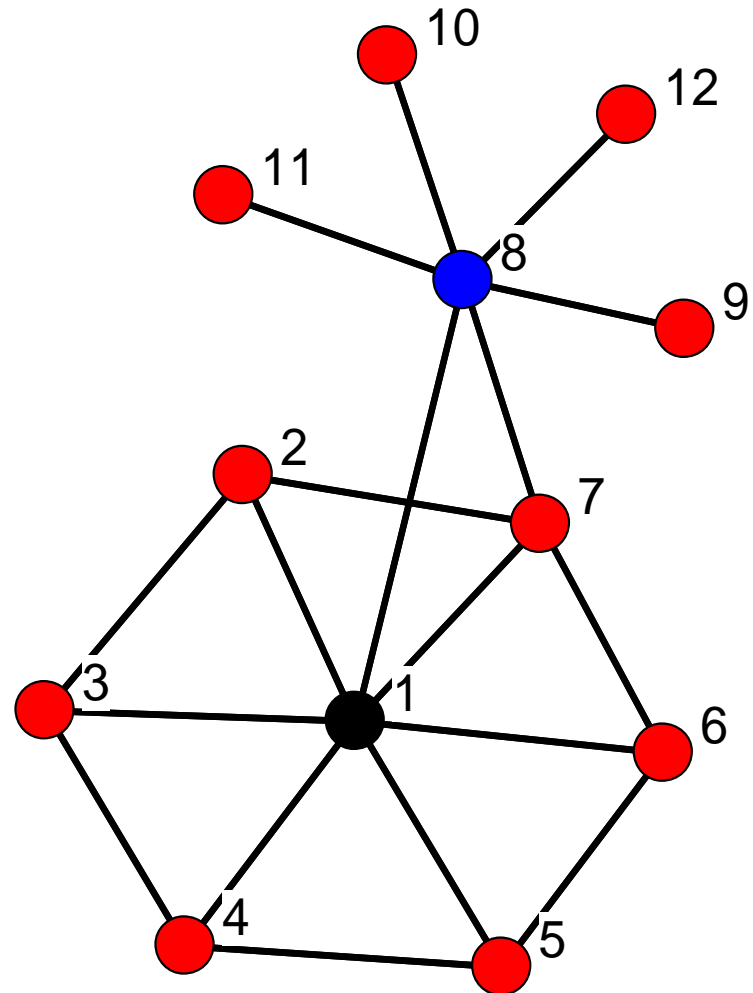
Proximity

	Jim	Jill	Jen	Joe
Jim	-	3	9	2
Jill	3	-	1	15
Jen	9	1	-	3
Joe	2	15	3	-



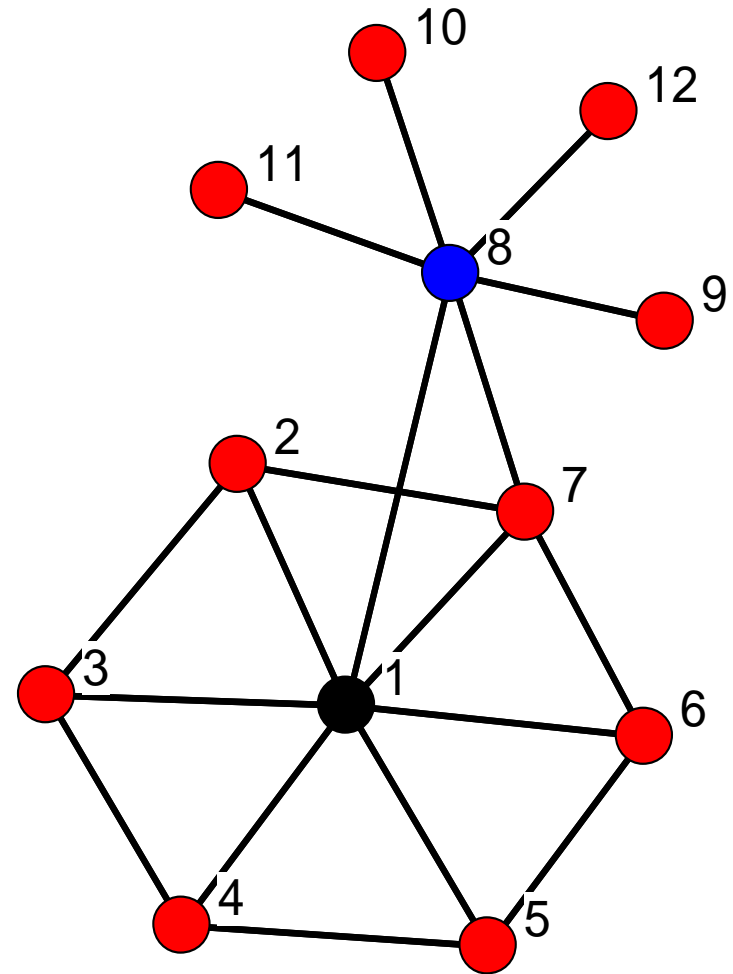
# Walks, Trails, Paths

- Path: can't repeat node  
1-2-3-4-5-6-7-8  
Not 1-7-2-1-8
- Trail: can't repeat line  
1-2-3-1-7-8  
Not 1-7-2-1-7-8
- Walk: unrestricted  
1-2-3-1-2-7-1-8



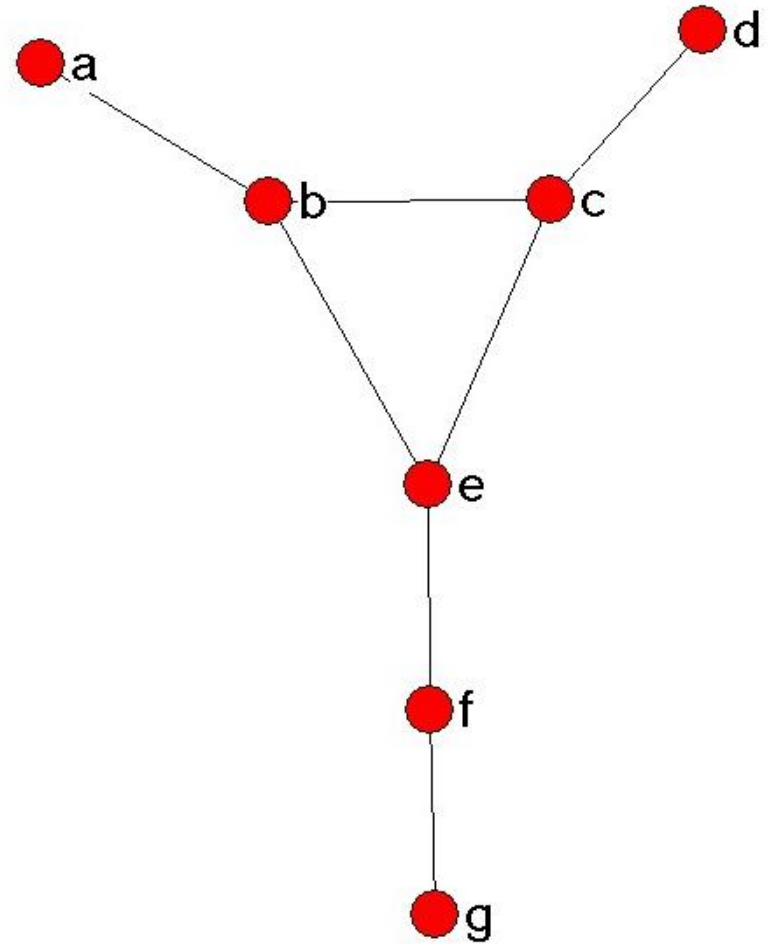
# Length & Distance

- Length of a path is number of links
- Distance between two nodes is length of shortest path (aka geodesic)
- 1-2-3-4-5-6-7-8 has length 7
- Distance from 1 to 8 is 1
- Distance from 4 to 9 is 3



# Geodesic Distance Matrix

	a	b	c	d	e	f	g
a	0	1	2	3	2	3	4
b	1	0	1	2	1	2	3
c	2	1	0	1	1	2	3
d	3	2	1	0	2	3	4
e	2	1	1	2	0	1	2
f	3	2	2	3	1	0	1
g	4	3	3	4	2	1	0

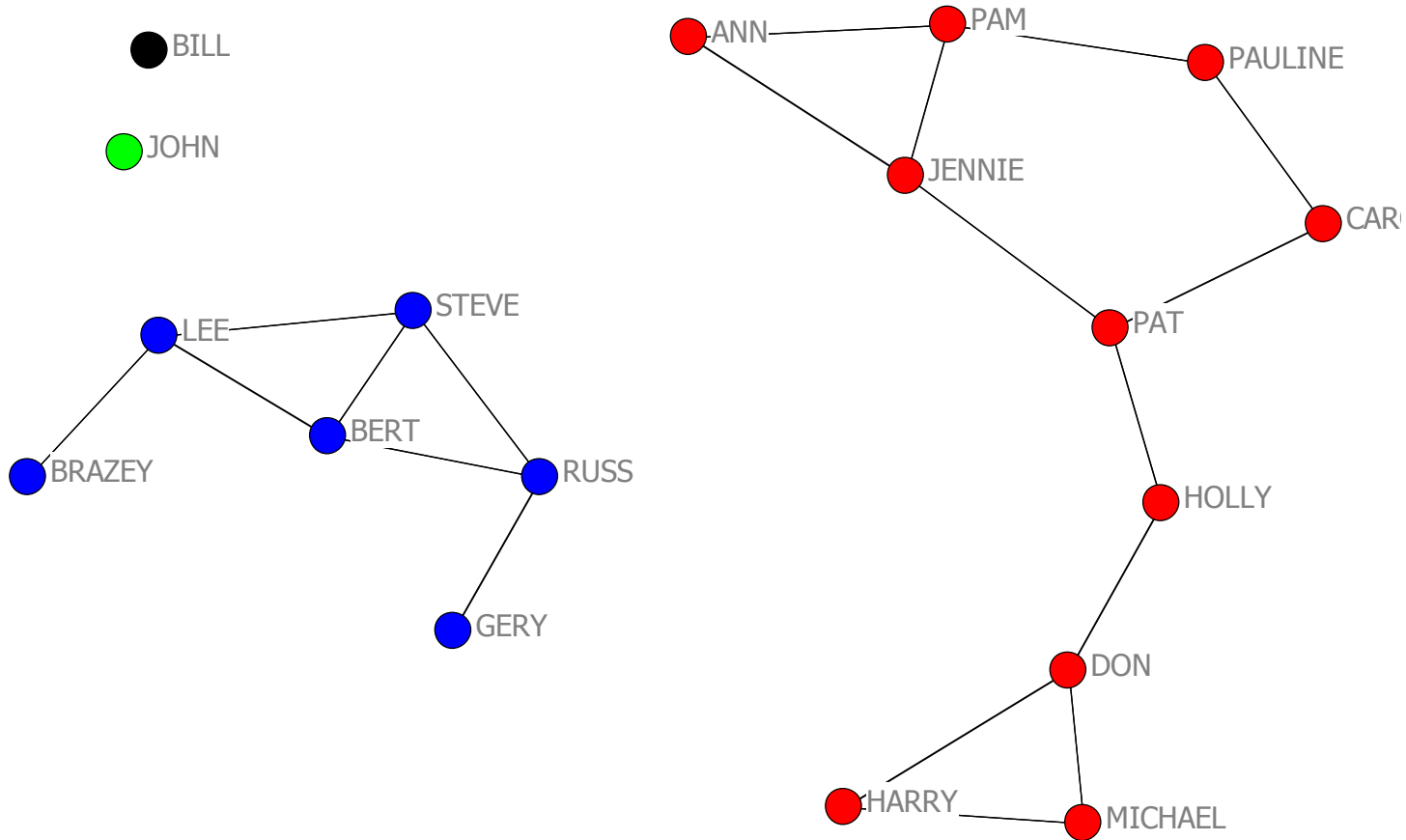


# Components

- Maximal sets of nodes in which every node can reach every other by some path (no matter how long)
- A connected graph has just one component

Relations form different networks. Components don't.

# A network with 4 components



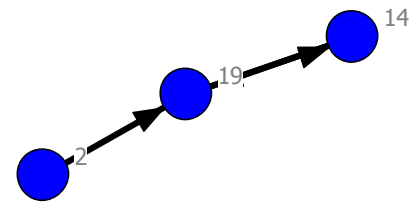
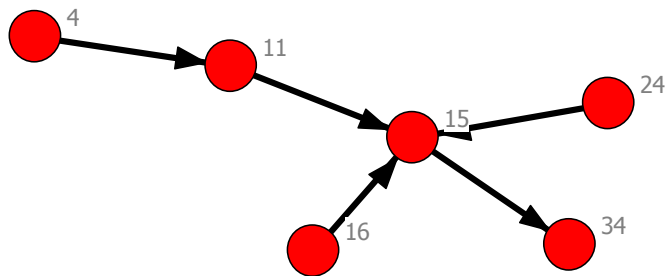
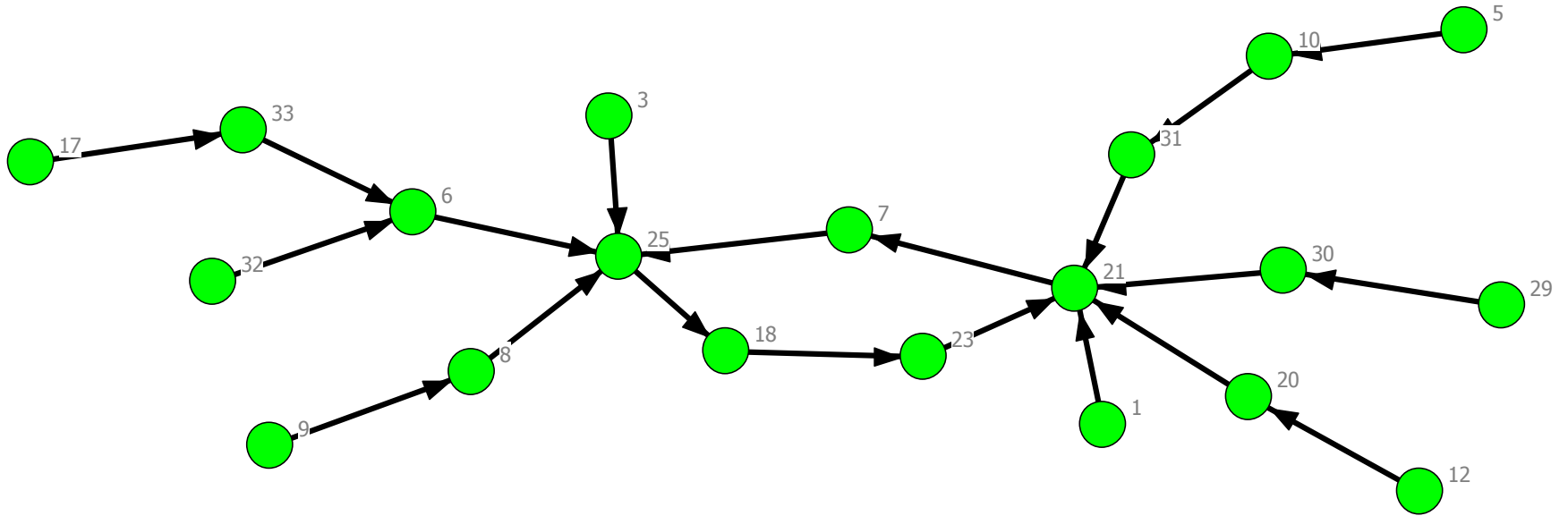
# Strong and Weak Components

Directed Graphs.

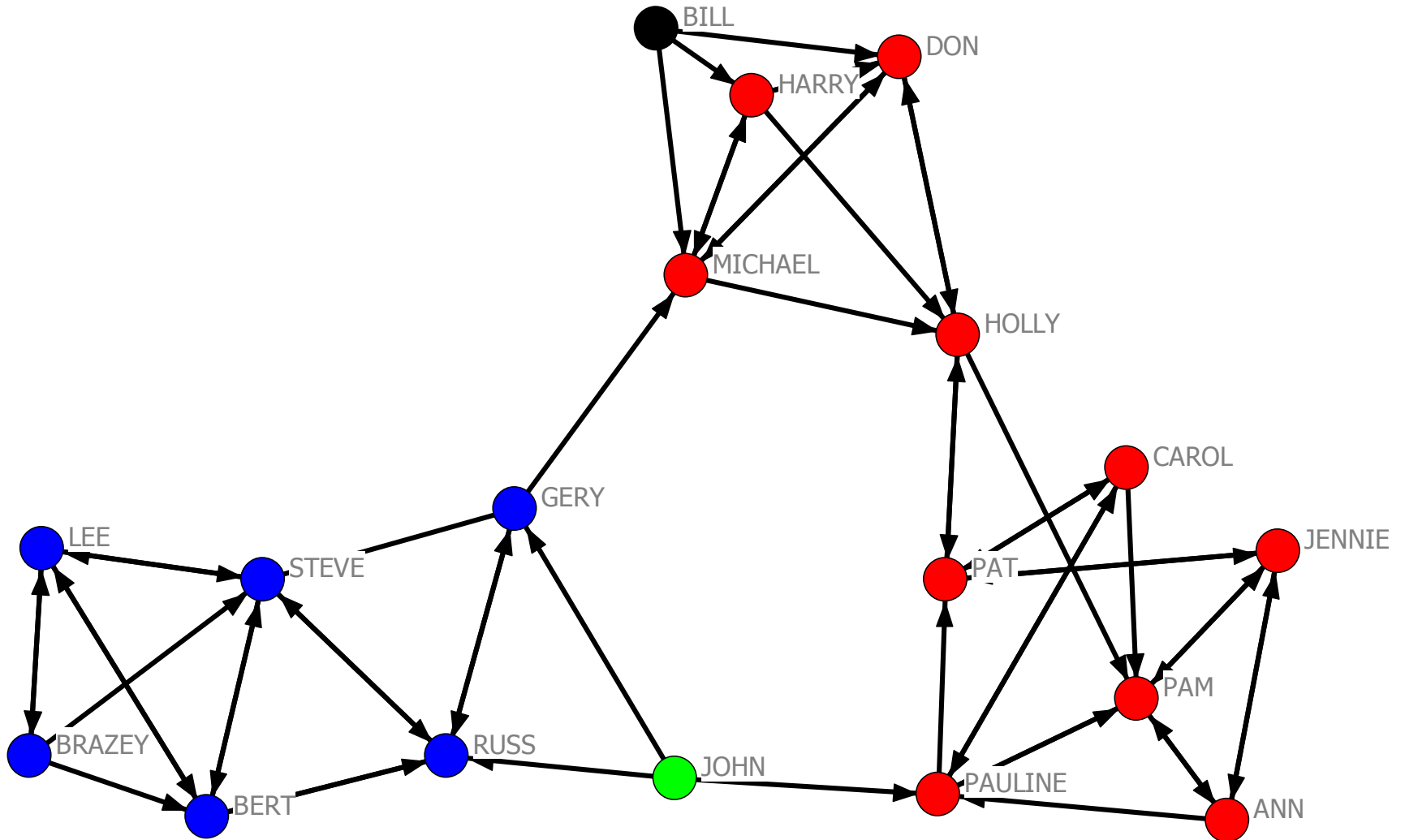
Weak component are components of undirected underlying network

Strong components maximal subgraphs in which there is a path from each actor to every other actor.

# A network with 3 weak components

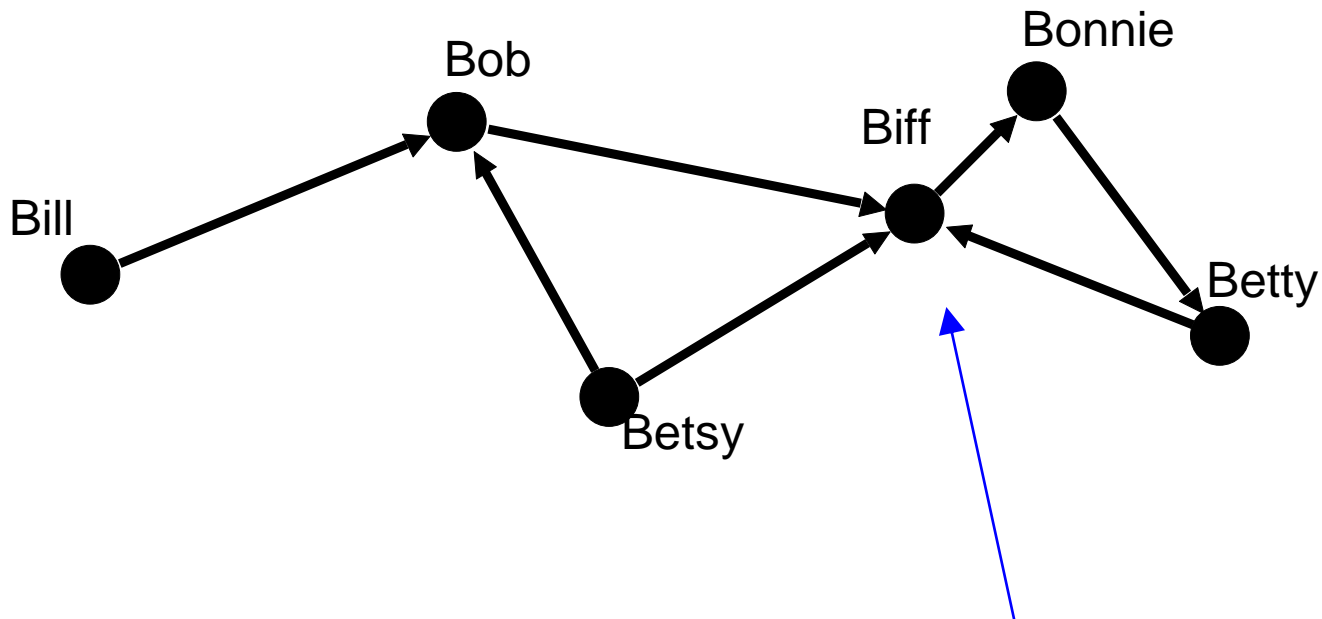


# Strong components



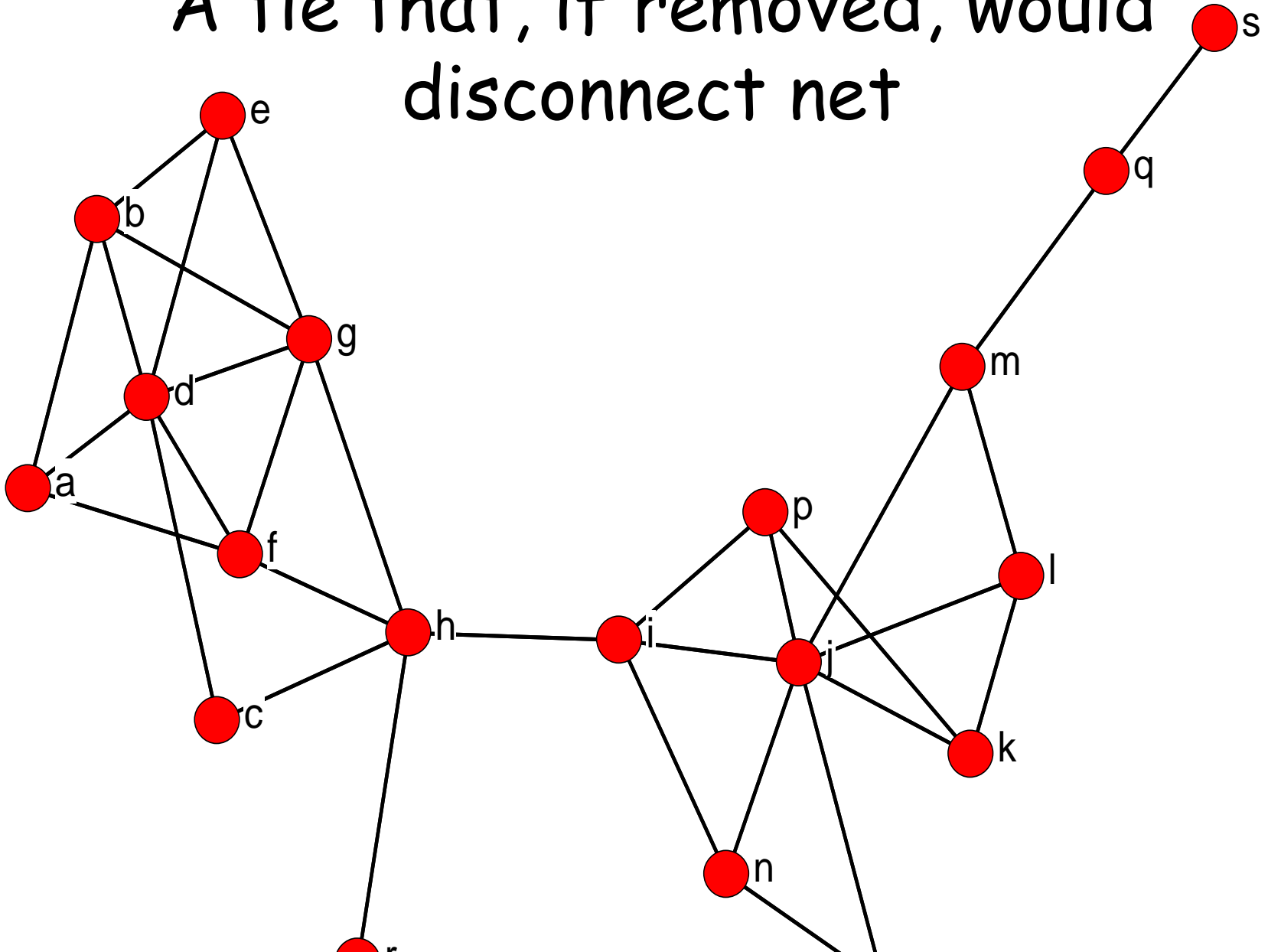
# Cutpoints

- Nodes which, if deleted, would increase the number of components



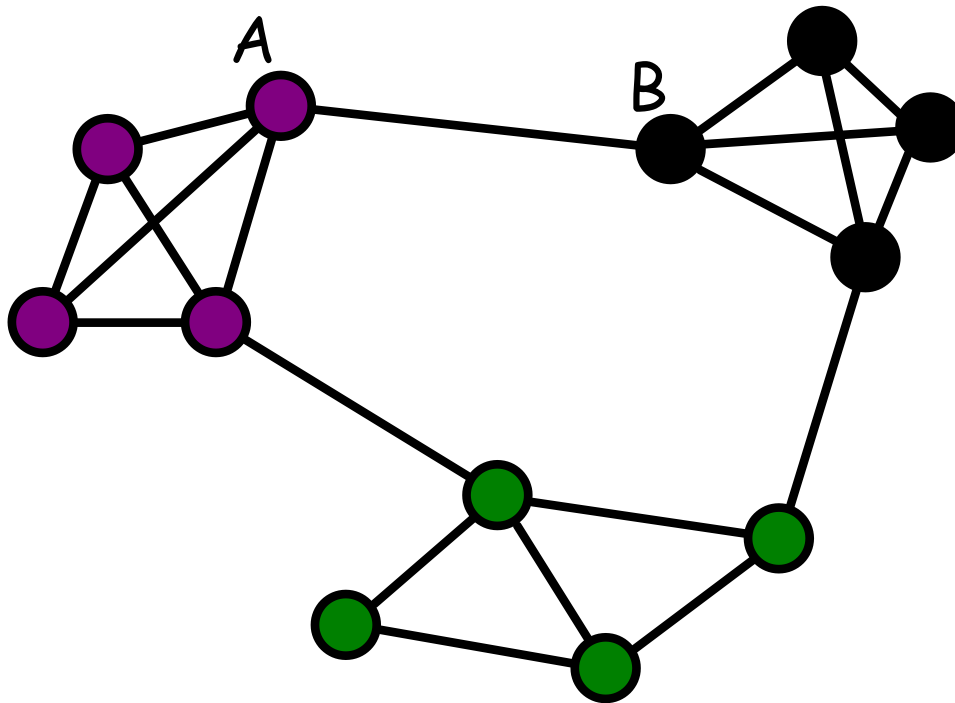
# Bridge

A tie that, if removed, would disconnect net



# Local Bridge of Degree K

- A tie that connects nodes that would otherwise be at least  $k$  steps apart



# Network Cohesion

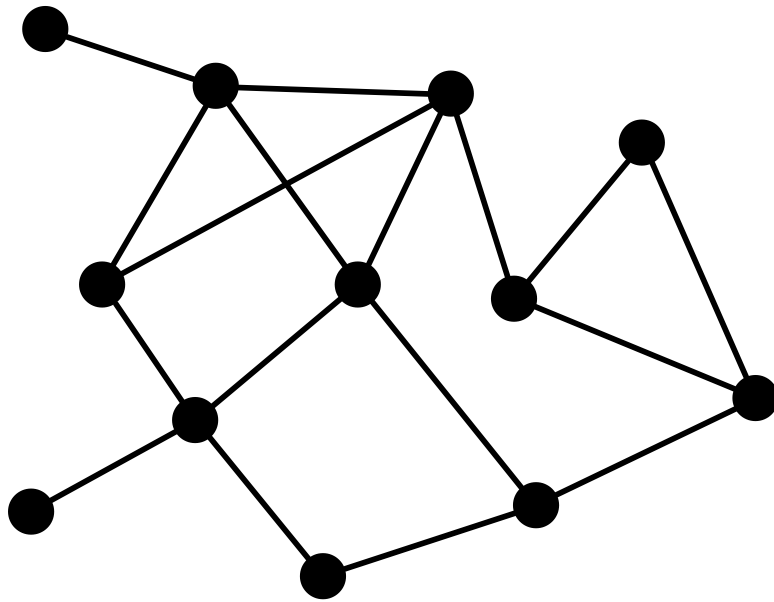
# Fragmentation

- Proportion of pairs of nodes that are unreachable from each other

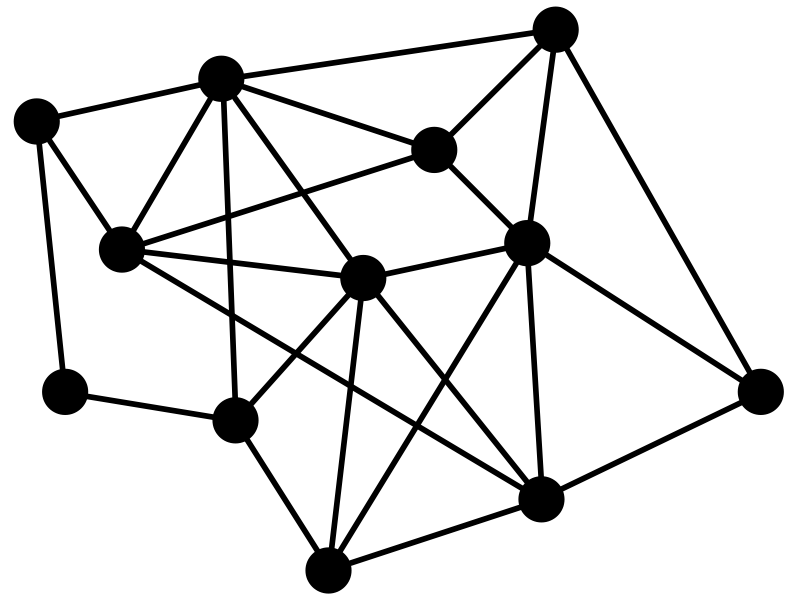
$$F = 1 - \frac{\sum_k s_k (s_k - 1)}{n(n - 1)}$$

# Density

- Number of ties, expressed as percentage of the number of ordered/unordered pairs



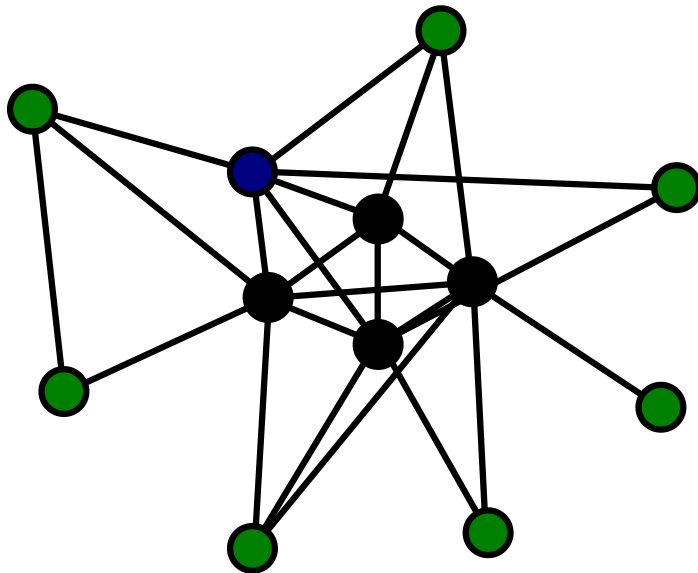
Low Density (25%)  
Avg. Dist. = 2.27



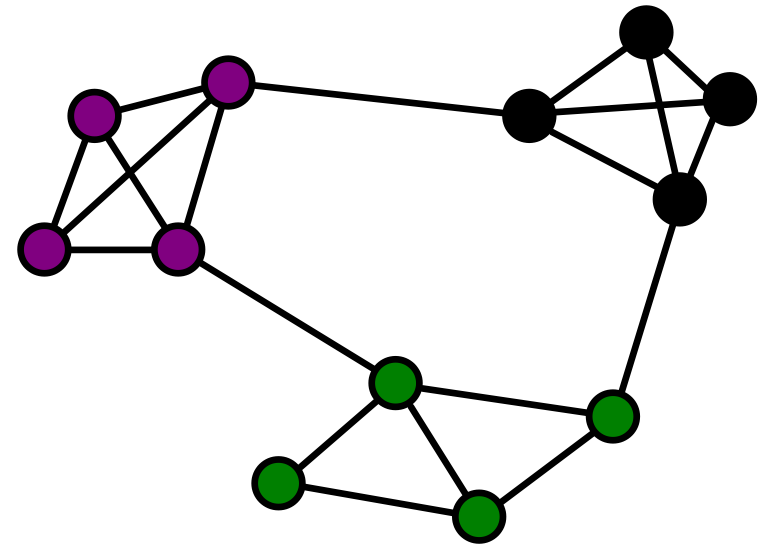
High Density (39%)  
Avg. Dist. = 1.76

# Average Distance

- Average distance between all pairs of nodes



Core/Periphery  
c/p fit = 0.97, avg. dist. = 1.9



Clique structure  
c/p fit = 0.33, avg. dist. = 2.4